Technical Notes

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Mixed Convection About a Cylinder Embedded to a Wedge in Porous Media

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Nomenclature

a = radius of the cylinder
b = constant, Eq. (1)
Gr = modified Grashof number, $g\beta K|T_w - T_0|a/\nu^2$
g = acceleration due to gravity
K = permeability
k = thermal conductivity
m = parameter associated with the wedge
Nu = Nusselt number, $q_w a/k(T_w - T_0)$
Pe = Peclet number, $U_0\, a/\alpha$
q = heat flux, $-k(\partial T/\partial y)_{y=0}$
Ra = modified Rayleigh number, $g\beta K|T_w - T_0|a/\alpha\nu$
Re = Reynolds number, $U_0 a/\nu$
T = temperature
u, v = velocity components in the x and y directions, respectively
U_0 = freestream velocity, $2ba^{m/(2-m)}/(2-m)$
W = complex potential, Eq. (1)
X, Y = linear orthogonal coordinates
x, y = curvilinear orthogonal coordinates
Z = complex variable
α = thermal diffusivity
β = coefficient of thermal expansion
γ, ϵ = buoyancy parameters
θ = dimensionless temperature
ξ, η = dimensionless coordinates
ν = kinematic viscosity
Φ = velocity potential
ψ = dimensionless stream function

Introduction

BECAUSE of its wide range of applicability in industrial operations, the mixed convection flow about a circular cylinder and a sphere immersed in a fluid-saturated porous medium has been studied in detail in recent years.[1-3] This Note deals for the first time with the mixed convection boundary layer about an isothermal circular cylinder that is embedded to an unheated wedge of included angle $m\pi$ and in a fluid-saturated porous medium. This is a more realistic model of geological formations than a sharp-edge configuration. The conservation equations describing the two-dimensional nonsimilar boundary-layer flow are transformed to a dimensionless form and then are solved numerically by a finite-difference method similar to that proposed by Keller.[4] Nusselt number results are presented over a wide range of values of the buoyancy force parameter and for various wedge angles.

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Analysis

Figure 1 shows the physical model to be considered here. A circular cylinder of radius a is attached to a wedge of included angle $m\pi$ with its horizontal axis coincident with the axis of the leading edge of the wedge, and this combined body is placed in a fluid-saturated porous medium. The freestream has velocity U_0 and temperature T_0. The wedge is maintained at the uniform temperature T_0, whereas the surface of the cylinder is maintained at a uniform temperature T_w, which may be greater or smaller than T_0. It is also assumed that the properties of the fluid and the porous medium are constant except for the density variation with temperature. A curvilinear orthogonal coordinate system Oxy is used, where x is the coordinate in the streamwise direction along the surface of the cylinder, measured from the stagnation point O, and y is the coordinate normal to the surface of the cylinder. The potential flow past the cylinder wedge in terms of complex potential $W(Z) = \Phi + i\psi$ as a function of complex variable $Z = X + iY$ is reported by Srivastava[5] as

$$W = -b\{\exp[i\pi(1-m)/(2-m)]Z^{2/(2-m)} + a^{4/(2-m)}\exp[-i\pi(1-m)/(2-m)]Z^{-2/(2-m)}\} \tag{1}$$

When $m = 0$, Eq. (1) gives the mixed convection flow about a circular cylinder immersed in a porous medium with the flow being uniform at infinity; $m = 1$ represents the mixed convec-

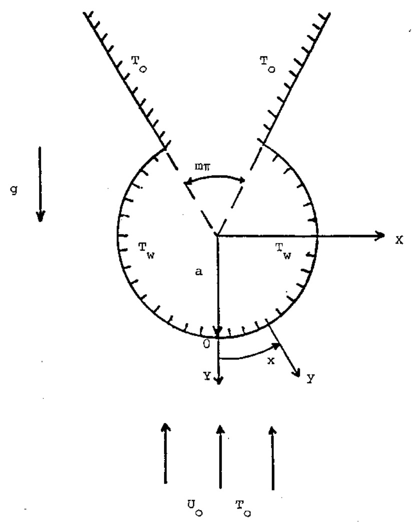

Fig. 1 Physical model and coordinate system.

tion flow about a semicircular cylinder embedded to a horizontal plane, the flow being no longer uniform at infinity. In this paper we study the boundary layer that forms on the cylinder when the wedge angle $m\pi$ is varied between $0 \leq m\pi < 2\pi$. In this respect, if we introduce the nondimensional variables ξ, η, ψ, and θ, where $\xi = x/a$, $\eta = Pe^{1/2}y/a$, $\bar{\psi} = \alpha Pe^{1/2}\psi$, and $\theta = (T - T_0)/(T_w - T_0)$, the governing boundary-layer equations, derived in Cheng,[1] can be written as

$$\frac{\partial^2 \psi}{\partial \eta^2} = \gamma \frac{\partial \theta}{\partial \eta} \sin\xi \tag{2}$$

$$\frac{\partial^2 \theta}{\partial \eta^2} = \frac{\partial \psi}{\partial \eta}\frac{\partial \theta}{\partial \xi} - \frac{\partial \psi}{\partial \xi}\frac{\partial \theta}{\partial \eta} \tag{3}$$

and are to be solved with the boundary conditions

$$\psi = 0, \quad \theta = 1 \text{ on } \eta = 0 \tag{4}$$

$$\frac{\partial \psi}{\partial \eta} \to 2\sin\left(\frac{2\xi}{2-m}\right), \quad \theta \to 0 \quad \text{as} \quad \eta \to \infty \tag{5}$$

In these equations $\gamma = g\beta K |T_w - T_0| / U_0 \nu = Gr/Re$ is a nondimensional parameter that describes the relative importance of natural convection to forced convection flow. For $\gamma > 0$, i.e., $T_w > T_0$ (heated cylinder), the buoyancy force assists the forced flow, whereas if $\gamma < 0$, i.e., $T_w < T_0$ (cooled cylinder), the buoyancy force opposes the forced flow.

Solution

Because of the boundary condition [Eq. (5)], the present problem does not possess a similarity solution as in the case of a free circular cylinder ($m = 0$).[1] Thus, we shall solve numerically these equations by using the Keller[4] box method. In this respect we introduce the appropriate transformation $\psi = \xi f(\xi, \eta)$ so that Eqs. (2) and (3) become

$$\frac{\partial^2 f}{\partial \eta^2} = \gamma \frac{\sin\xi}{\xi}\frac{\partial \theta}{\partial \eta} \tag{6}$$

$$\frac{\partial^2 \theta}{\partial \eta^2} + f\frac{\partial \theta}{\partial \eta} = \xi\left(\frac{\partial f}{\partial \eta}\frac{\partial \theta}{\partial \xi} - \frac{\partial f}{\partial \xi}\frac{\partial \theta}{\partial \eta}\right) \tag{7}$$

and the boundary conditions [Eqs. (4) and (5)] now read

$$f = 0, \quad \theta = 1 \quad \text{at} \quad \eta = 0 \tag{8}$$

$$\frac{\partial f}{\partial \eta} \to 2\sin\left(\frac{2\xi}{2-m}\right)/\xi \quad \theta \to 0 \quad \text{as} \quad \eta \to \infty \tag{9}$$

The equations for $f_0(\eta)$ and $\theta_0(\eta)$ that are the values of f and θ at the stagnation point O where $\xi = 0$, take the form

$$f_0'' = \gamma\theta_0' \tag{10}$$

$$\theta_0'' + f\theta_0' = 0 \tag{11}$$

with the boundary conditions

$$f_0 = 0, \qquad \theta_0 = 1 \qquad \text{at} \qquad \eta = 0 \tag{12}$$

$$f_0' \to 4/(2-m), \qquad \theta_0 \to 0 \qquad \text{as } \eta \to \infty \tag{13}$$

Here, primes denote differentiation with respect to η. If we further put

$$f_0 = 2(2-m)^{-1/2}\bar{f}(\bar{\eta}),\ \bar{\eta} = 2(2-m)^{-1/2}\eta,\ \epsilon = (2-m)\gamma/4 \tag{14}$$

then Eqs. (10–13) reduce to

$$\bar{f}''' + \bar{f}\bar{f}'' = 0 \tag{15}$$

$$\bar{f}(0) = 0 \qquad \bar{f}'(0) = 1 + \epsilon, \qquad \bar{f}'(\infty) = 1 \tag{16}$$

In these equations primes now designate differentiation with respect to $\bar{\eta}$. It is worth pointing out that Eq. (15) is identical to the equations obtained by Merkin[6] in the problem of mixed convection along a vertical semi-infinite flat plate in a porous medium. From the temperature profiles thus calculated, we can find the heat transfer from the cylinder, which in terms of the Nusselt number is given by

$$NuPe^{-1/2} = -\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0} \tag{17}$$

Results and Discussion

The system of coupled differential equations [Eqs. (6) and (7)] along with the boundary conditions [Eqs. (8) and (9)] were solved numerically using the Keller box method for a wide range of values of γ and m. Figures 2 show the variation of the Nusselt number as a function of ξ for $\gamma = -2, -1, 0$ (forced convection), 1, 5, and 10, and $m = 0$, 1/2, and 1. It is observed that the heat transfer parameter increases with an increase of the buoyancy parameter γ (for a fixed value of m) and decreases with an increase of the wedge parameter m (for a fixed value of γ). Furthermore, for an assisting flow ($\gamma > 0$), solutions for this problem can be found for all values of γ and m (<2).

For the special case of a complete cylindner ($m = 0$), the present problem has been treated by Cheng,[1,7] who obtained

$$NuPe^{-1/2} = \sqrt{2}\left[-\theta'(0)\right]\cos\frac{\xi}{2} \tag{18}$$

where the value of $[-\theta'(0)]$ as a function of γ is tabulated. The numerical results presented in the present paper are graphically indistinguishable from those obtained using expression (17).

For $\gamma = 0$ (force convection flow) Eqs. (2) and (3) with the boundary conditions [Eqs. (4) and (5)] can be intergrated to give

$$\psi = 2\eta\sin[2\xi/(2-m)] \tag{19}$$

$$\theta = \text{erfc}\{(2\eta/\sqrt{4-2m})\cos[\xi/(2-m)]\} \tag{20}$$

Hence,

$$NuRa^{-1/2} = (2/\sqrt{\pi})(2/\sqrt{4-2m})\cos[\xi/(2-m)] \tag{21}$$

where erfc( ) is the complementary error function. Again, the numerical results presented in this paper are graphically indistinguishable from those obtained using Eq. (20).

It is interesting to note that the solutions (17) and (20) are such that $NuPe^{-1/2} = 0$ when $\xi = (2-m)\pi/2$. The numerical prediction of the value of ξ at which the heat transfer is zero is correct to within less than 1% for all values of m considered.

For opposing flows ($\gamma < 0$), the problem is similar, in some respects, to that considered by Merkin,[6] and there are several important points that should be noted. First, when $\gamma(2-m)/4 < -1.354$, there are no solutions of the present problem near the lower stagnation point of the cylinder, i.e., $\xi = 0$. Also, for m and γ in the range $-1.354 < \gamma(2-m)/4 < -1$, dual solutions occur at $\xi = 0$. Both of these branches are such that $\bar{f}'(0) < 0$; thus, we expect that solutions of Eqs. (6) and (7) cannot be obtained for any $\xi > 0$, since a region of reversed flow, i.e., $\partial\psi/\partial\eta < 0$, develops near the wall. Finally, we should mention that, for m and γ in the range $-1 < \gamma(2-m)/4 < 0$, numerical solutions of the governing Eqs. (6) and (7) may be obtained only up to a certain value of ξ,

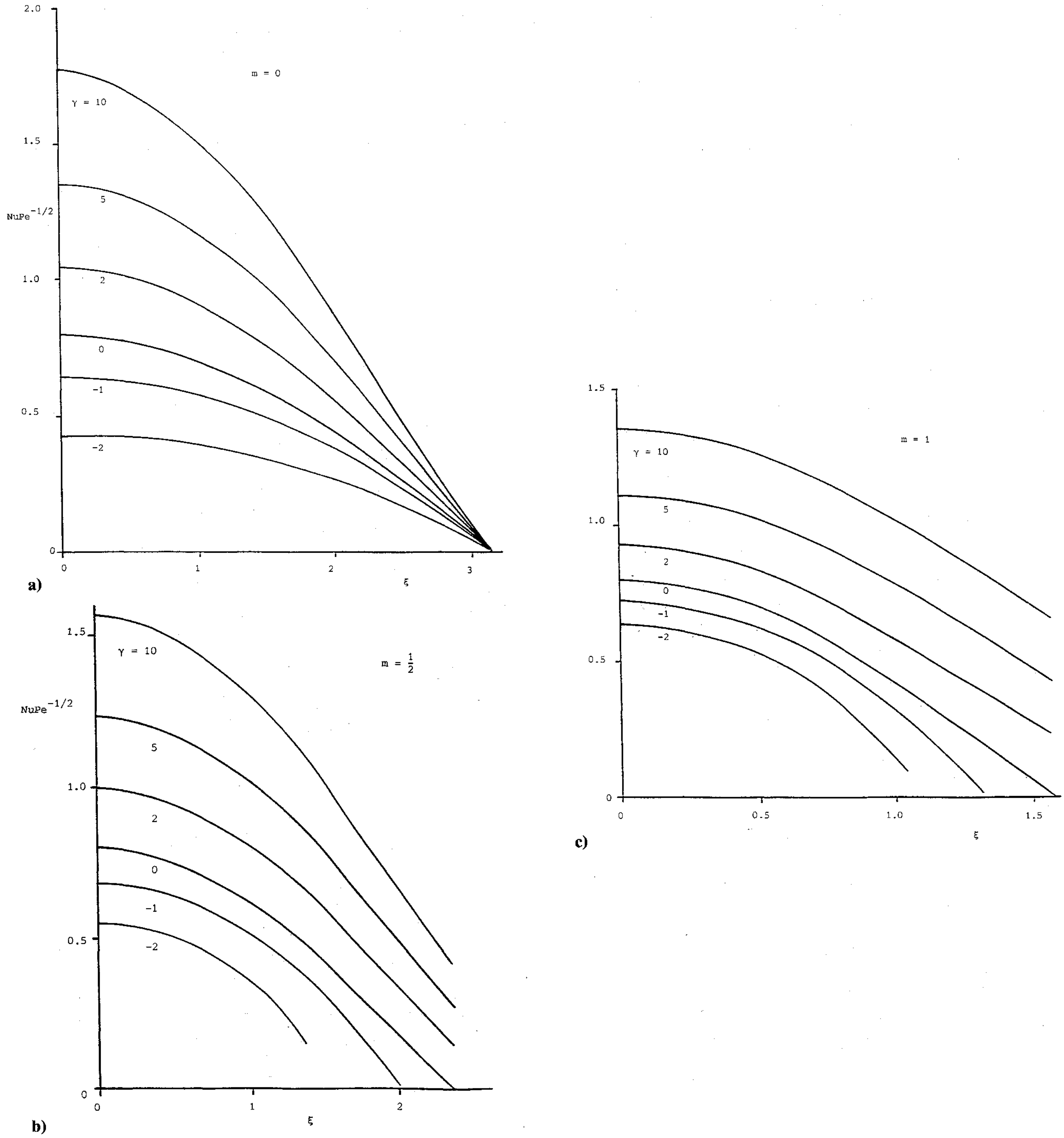

Fig. 2 Variation of the Nusselt number as a function of ξ for $\gamma = -2, -1, 0, 2, 5$, and 10 and for a) $m = 0$, b) $m = 0.5$, and c) $m = 1$.

namely, where $\partial\psi/\partial\eta < 0$ at the wall. Beyond this point the present numerical scheme is no longer appropriate. We can estimate this particular value of ξ from the condition that reversed flow occurs (i.e., $\partial\psi/\partial\eta < 0$, at $\eta = 0$). From this it follows that

$$(\gamma/2)\sin\xi + \sin[2\xi/(2-m)] < 0 \tag{22}$$

For specified values of m and γ, the value of ξ predicted by Eq. (21) agrees with the values of ξ as given in Fig. 2.

Acknowledgments

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Green's Function Solution to Radiative Heat Transfer Between Longitudinal Gray Fins

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Nomenclature

b	= fin thickness, $=2t$
$f(x_0)$	= minimum dimensional location on adjacent fin where radiation is incident
$f^*(\eta)$	$= f(x_0)/l$
$G(\eta,\eta_0)$	= Green's function; Eq. (6)
k_0	= thermal conductivity at reference temperature T_b
l	= total length of fin and tube radius, $= w + R$
N_c	= conduction-radiation number, $l^2\sigma T_b^3/(k_0 t)$
$q_{0,x_0}(x_0)$	= dimensional radiosity
$Q_0(\eta)$	= dimensionless radiosity, $Q_0 = q''_{0,x_0}/(\sigma T_b^4)$
R	= tube radius
$T(x_0)$	= dimensional temperature
T_b	= dimensional tube-base temperature
t	= fin half-thickness, $= b/2$
w	= fin length
α	= angle
β_m	= dimensional coefficients required for temperature-dependent thermal conductivity
β_m^*	= dimensionless Taylor series coefficients, $\beta_m T_b^m$
$\delta(\eta_0 - \eta)$	= Dirac delta function
ϵ	= emissivity
γ	= opening angle between adjacent fins
η	= dimensionless spatial variable, x_0/l
η_0	= dimensionless "dummy" spatial variable
η^*	= dimensionless spatial variable, $\eta^* = (\eta - \tau)/(1-\tau)$
$\theta(\eta)$	= dimensionless temperature T/T_b
$\kappa^*(\eta)$	= cutoff angle shown in Fig. 1
λ	= dimensionless half-thickness of fin to entire length of radiator, t/l
ξ	= dimensionless spatial variable, x_1/l
ρ	= reflectivity
σ	= Stefan-Boltzmann constant
τ	= dimensionless radius to radiator length ratio, R/l

Introduction

OWING to the nonlinear and rather complex nature associated with conductive-radiative heat transfer, the development of new computational tools that can readily and accurately solve this class of problem is salient. The objective of the present study is to demonstrate the applicability and versatility of a pure integral formulation for radiative-conductive heat transfer problems. Preliminary results indicate that this formulation permits an accurate, fast, and stable computational procedure to be implemented. A multidimensional extension of the one-dimensional methodology described within this Note is presently underway.

Several idealized one-dimensional, steady-state fin radiator studies have been expounded upon since the 1960s.[1-5] However, most previous investigations[1-7] have had several shortcomings with regard to their assumptions (no base interactions or solar sources), analysis, and solution. The inclusion of base interactions on a longitudinal fin array was addressed by Schnurr.[7] However, Schnurr presented a limited set of results and did not investigate the effect of a temperature-dependent thermal conductivity. Sparrow and Eckert[2] and Sarabia[5] did include base interactions in the study of tube-sheet radiators.

Analysis

Figure 1 illustrates the physical situation under consideration. The dimensionless tube radius is denoted by τ, and the dimensionless fin length is $(1-\tau)$. A black tube is maintained at the uniform dimensionless temperature of unity, and the ambient temperature is taken as 0 K. The straight, opaque fins are assumed to behave as diffuse gray emitters. Inclusion of a solar source would not affect the analytic-numeric method, though the gray assumption would then become questionable. In this study, a generalized temperature-dependent thermal conductivity[8] has also been included in the analysis. Finally, the fins are assumed to be thin in order to ensure the one-dimensional assumption in the radial direction.

Primitive Formulation

The analytic formulation adopted in this Note appeals to the primitive formulation developed in Refs. 9 and 10, which preserves the direct coupling between the temperature and radiosity. Using the dimensionless quantities shown in the Nomenclature, one can write the dimensionless heat equation as

$$\frac{\mathrm{d}^2\theta(\eta)}{\mathrm{d}\eta^2} = -\sum_{m=1}^{M_{\max}} \frac{\beta_m^*}{(m+1)}\frac{\mathrm{d}^2}{\mathrm{d}\eta^2}[\theta(\eta)-1]^{m+1} + Q_0(\eta)N_c - N_c\int_{\xi=f^*(\eta)}^{1} Q_0(\xi)K_1^*(\eta,\xi)\,\mathrm{d}\xi - N_c\int_{\alpha=0}^{\kappa^*(\eta)} K_2^*(\eta,\alpha)\,\mathrm{d}\alpha \tag{1a}$$

where the first term on the right side of Eq. (1a) accounts for the nonlinear contributions of the temperature-dependent thermal conductivity[8] (note that $\beta_0^* = 1$). The shape factors for the surface radiation terms can be expressed as

$$K_1^*(\eta,\xi) = \frac{\sin^2\gamma}{2}\frac{\eta\xi}{(\eta^2+\xi^2-2\eta\xi\cos\gamma)^{3/2}} \tag{1b}$$

$$K_2^*(\eta,\alpha) = \frac{\tau^2\sin\alpha}{2}\left[\frac{\eta\cos\alpha-\tau}{(\tau^2+\eta^2-2\eta\tau\cos\alpha)^{3/2}}\right] \tag{1c}$$

Note that the second integral vanishes as τ goes to 0. This case was considered by Sparrow et al.[1] When this occurs, it is

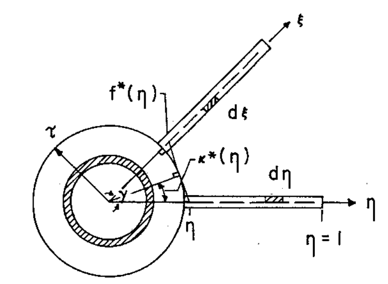

Fig. 1 Dimensionless coordinate system displaying fin configuration.

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